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Quantum PBR Theorem as a Monty Hall Game

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Abstract: The quantum Pusey–Barrett–Rudolph (PBR) theorem addresses the question of whether the quantum state corresponds to a ψ-ontic model (system's physical state) or to a ψ-epistemic model (observer's knowledge about the system). We reformulate the PBR theorem as a Monty Hall game and show that winning probabilities, for switching doors in the game, depend on whether it is a ψ-ontic or ψ-epistemic game. For certain cases of the latter, switching doors provides no advantage. We also apply the concepts involved in quantum teleportation, in particular for improving reliability.

1. Introduction

No-go theorems in quantum foundations are vitally important for our understanding of quantum physics. Bell's theorem [1] exemplifies this by showing that locally realistic models must contradict the experimental predictions of quantum theory.

There are various ways of viewing Bell's theorem through the framework of game theory [2]. These are commonly referred to as nonlocal games, and the best known example is the CHSHgame; in this scenario, the participants can win the game at a higher probability with quantum resources, as opposed to having access to only classical resources. There has also been work on the relationship between Bell's theorem and Bayesian game theory [3]; in a subset of cases, it was shown that quantum resources provide an advantage, and lead to quantum Nash equilibria. In [4], it was shown that quantum nonlocality can outperform classical strategies in games where participants have conflicting interests. In [5], a nonlocal game was constructed where quantum resources did not offer an advantage.

Beyond Bell's theorem, entropic uncertainty relations can be viewed in the framework of a guessing game [6,7]; the uncertainty relation constrains the participant's ability to win the game. More broadly, the relationship between quantum theory and game theory was investigated in [8–10]. The Monty Hall game [11–14] has also been generalized into quantum versions [15–21].

The Pusey–Barrett–Rudolph (PBR) theorem [22] is a relatively recent no-go theorem in quantum foundations. It addresses the question of whether the quantum state corresponds to a ψ-ontic model (physical state of a system) or to a ψ-epistemic model (observer's knowledge about the system) [23]. Notable developments of the PBR theorem and ψ-epistemic models were carried out in [24–32], including on the issue of quantum indistinguishability [33–35], as well as being interpreted through the language of communication protocols [36,37].

Analogous to the game formulation of Bell's theorem, a desirable construction is to view the PBR theorem through the lens of a game. One instantiation of this is in an exclusion game where the participant's goal is to produce a particular bit string [38,39]; this has been shown to be related to the task of quantum bet hedging [40]. Furthermore, concepts involved in the PBR proof have been used for a particular guessing game [41].

In this article, we reformulate the PBR theorem into a Monty Hall game. This particular gamification of the theorem highlights that winning probabilities, for switching doors in the game, depend on whether it is a ψ-ontic or ψ-epistemic game; we also show that in certain ψ-epistemic games, switching doors provides no advantage. This may have consequences for an alternative experimental test of the PBR theorem. Furthermore, we shall also use the concepts involved to modify quantum teleportation [42,43] to view it as a Monty Hall game. Using these notions, we develop an error-correcting strategy for unreliable teleportation, which may be relevant for practical quantum networks.

2. PBR Theorem

We provide a rough sketch of the PBR proof [22] and highlight crucial outcomes. Two quantum systems are prepared independently, and each system is prepared in either state $|0\rangle$ or state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. This means that the total system is in one of the four possible non-orthogonal quantum states:

$$
\begin{aligned}
|\Psi_1\rangle &= |0\rangle \otimes |0\rangle, \quad |\Psi_2\rangle = |0\rangle \otimes |+\rangle, \\
|\Psi_3\rangle &= |+\rangle \otimes |0\rangle, \quad |\Psi_4\rangle = |+\rangle \otimes |+\rangle.
\end{aligned}
\tag{1}
$$

The total system is brought together and measured in the following entangled basis:

$$
\begin{aligned}
|\Phi_1\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle), \\
|\Phi_2\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle), \\
|\Phi_3\rangle &= \tfrac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle), \\
|\Phi_4\rangle &= \tfrac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle),
\end{aligned}
\tag{2}
$$

where $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$.

Invoking the Born probabilities, $|\langle\Phi_i|\Psi_h\rangle|^2$, where $i,h = 1,2,3,4$, we have for $i = h$, $|\langle\Phi_i|\Psi_i\rangle|^2 = 0$. This means that for any value i, the outcome $|\Phi_i\rangle$ never occurs when the system is prepared in quantum state $|\Psi_i\rangle$. The PBR proof showed that in ψ-epistemic models, there is a non-zero probability q (whose value does not need to be specified) that outcome $|\Phi_i\rangle$ occurs when state $|\Psi_i\rangle$ is prepared, thereby contradicting the predictions of quantum theory; hence, one can infer that the quantum state corresponds to a ψ-ontic model.

3. Classic Monty Hall

A character named Monty hosts a game show. There are three closed doors respectively labelled $\{1,2,3\}$. There is a prize behind one door and goats behind the remaining two. The prize door is denoted A_i where i takes one of the door labels, and this choice of prize door is made by the producers of the show. We assume in the game that when a random choice needs to be made, all options are chosen with the same probability. Hence, we have $P(A_i) = 1/3$ for all values i. The contestant on the show, who does not know which door the prize is behind, gets to pick a door; we label this as B_j where j takes door labels; given this is a random choice, we have $P(B_j|A_i) = 1/3$, for all values i,j. Next, Monty, who knows where the prize is, has to open a goat door, C_k, where k takes one of the door labels. Monty's decision is constrained through the game rule that he cannot open the door chosen by the contestant. Hence, we have the following conditional probabilities:

$$
P(C_k \,|\, B_j \cap A_i) = \begin{cases} \frac{1}{2}, & \text{if } i = j \neq k, \\ 1, & \text{if } i \neq j \neq k, \\ 0, & \text{otherwise.} \end{cases}
\tag{3}
$$

Once a goat door is opened, Monty offers the contestant the option to stick with the original choice or switch to the other unopened door. By sticking, the contestant's probability of opening the prize door is 1/3. Counter-intuitively, by switching doors, the probability of winning increases to 2/3. This can be seen by computing the non-zero joint probabilities for all events:

$$P(A_i \cap B_j \cap C_k) = P(C_k \mid B_j \cap A_i)P(B_j|A_i)P(A_i), \tag{4}$$

and then summing those values for the events where the contestant would win by switching. This results in:

$$P(\text{win if switch}) = \sum_{i \neq j \neq k} P(A_i \cap B_j \cap C_k) = \frac{2}{3}. \tag{5}$$

4. Ignorant Monty Hall

Just as in the classic case, we have $P(A_i) = 1/3$ and $P(B_j|A_i) = 1/3$, for all values i, j. However, in this game, Monty does not know what lies behind any of the doors. The only constraint is that Monty cannot open the door chosen by the contestant; hence, we have:

$$P(C_k \mid B_j \cap A_i) = \begin{cases} 0, & \text{if } j = k, \\ \frac{1}{2}, & \text{otherwise.} \end{cases} \tag{6}$$

There is now a probability that he will open up the prize door by accident, thus ending the game:

$$P(\text{opens prize door}) = \sum_{i=k \neq j} P(A_i \cap B_j \cap C_k) = \frac{1}{3}. \tag{7}$$

This implies that the probability that he opens a goat door is 2/3. The joint probability that Monty opens a goat door and the contestant wins by switching doors can be computed to be 1/3. From the last two values, we can calculate the conditional probability:

$$P(\text{win if switch} \mid \text{opens goat door}) = \frac{1/3}{2/3} = \frac{1}{2}. \tag{8}$$

This means if Monty opens a goat door, then the contestant's probability of winning is the same whether the contestant chooses to switch the door or not.

5. ψ-Ontic Monty Hall Game

Antidistinguishability [30,44,45], where there is a measurement for which each outcome identifies that a specific member of a set of quantum states was definitely not prepared, is highlighted in the PBR proof by $|\langle \Phi_i | \Psi_i \rangle|^2 = 0$ for all i. We will exploit this to construct our game, which can be thought of as a quantum Ignorant Monty Hall game.

For state $|\Psi_1\rangle$ in (1), we have:

$$\begin{aligned} |\langle \Phi_1 | \Psi_1 \rangle|^2 &= 0, & |\langle \Phi_2 | \Psi_1 \rangle|^2 &= 1/4, \\ |\langle \Phi_3 | \Psi_1 \rangle|^2 &= 1/4, & |\langle \Phi_4 | \Psi_1 \rangle|^2 &= 1/2. \end{aligned} \tag{9}$$

For the other states in (1), the same probability distribution $(0, 1/4, 1/4, 1/2)$ occurs, but across the different outcomes (2); hence, we will focus our game on $|\Psi_1\rangle$, but similar constructions hold for the other states.

The Monty Hall gamification is as follows: There are four doors labelled $\{1, 2, 3, 4\}$, and these correspond to the different measurement outcomes listed in (2). The prize door A_i, where i takes one of the door labels, is the outcome $|\Phi_i\rangle$ that the state $|\Psi_1\rangle$ collapses to upon measurement. For a ψ-ontic game, through the Born probabilities (9), we have $P(A_i) = |\langle \Phi_i | \Psi_1 \rangle|^2$.

The contestant on the show does not know what state from (1) is used and is only aware of the possible measurement outcomes (2). Based on this limited information, the contestant randomly picks one of the doors, which we denote B_j where j is the corresponding door label; hence, we have $P(B_j|A_i) = 1/4$, for all values i, j.

Monty's decision corresponds to the predictions of quantum theory. He is aware that state $|\Psi_1\rangle$ was used and has access to the Born probabilities (9). The door opened by Monty is denoted C_k where k is one of the door labels. The main insight to construct this game is that when Monty opens a goat door, he is opening a door that has probability zero of having a prize in it. For our game, a door that definitely does not have a prize in it corresponds to outcome $|\Phi_1\rangle$ as $P(A_1) = |\langle\Phi_1|\Psi_1\rangle|^2 = 0$. Hence, in this game, Monty will open door C_1 unless the contestant has already chosen this door as his/her pick (as Monty cannot open the door chosen by the contestant); in that case, Monty will open one of the other remaining doors with equal probability, and there is a chance he may open up the prize door as in the Ignorant Monty Hall game. From these factors, one can compute,

$$P(C_k \,|\, B_j \cap A_i) = \begin{cases} \frac{1}{3}, & \text{if } j = 1 \text{ and } k = 2, 3, 4, \\ 1, & \text{if } j \neq 1 \text{ and } k = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

The probability that Monty opens the prize door is:

$$P(\text{opens prize door}) = \sum_{i=k\neq j} P(A_i \cap B_j \cap C_k) = \frac{1}{12}. \tag{11}$$

This implies that the probability that he opens a goat door is $11/12$. Monty then offers the option to stick or switch. Suppose the contestant always sticks with the initial choice. Then, the probability of winning if sticking and Monty opening a goat door is:

$$\sum_{i=j\neq k} P(A_i \cap B_j \cap C_k) = \frac{1}{4}. \tag{12}$$

With that, we can compute the conditional probability:

$$P(\text{win if stick} \,|\, \text{opens goat door}) = \frac{1/4}{11/12} = \frac{3}{11}. \tag{13}$$

Suppose the contestant decides to always switch to one of the other two unopened doors with equal probability $1/2$. Let $|\Phi_l\rangle$ be the outcome switched to, and let D_l be the corresponding door. With that, we can compute $P(A_i \cap B_j \cap C_k \cap D_j) = P(D_l \,|\, C_k \cap B_j \cap A_i)P(C_k \,|\, B_j \cap A_i)P(B_j|A_i)P(A_i)$. Hence, the probability of winning if switching and Monty opening a goat door is:

$$\sum_{i=l\neq j\neq k} P(A_i \cap B_j \cap C_k \cap D_j) = \frac{1}{3}. \tag{14}$$

From that, one can calculate:

$$P(\text{win if switch} \,|\, \text{opens goat door}) = \frac{1/3}{11/12} = \frac{4}{11}. \tag{15}$$

In a ψ-ontic game, switching provides an advantage.

6. ψ-Epistemic Monty Hall Game

In the PBR proof, for the ψ-epistemic model, there is a non-zero probability q that outcome $|\Phi_1\rangle$ occurs when state $|\Psi_1\rangle$ is prepared. This implies that in a ψ-epistemic game, $P(A_1) = q \neq 0$. To allow

for a comparison with the ψ-ontic game, let $q = q_1 + q_2 + q_3$, and with that, let the other prize door probabilities take values $P(A_2) = (1/4) - q_1$, $P(A_3) = (1/4) - q_2$, and $P(A_4) = (1/2) - q_3$.

As in the ψ-ontic game, $P(B_j | A_i) = 1/4$, for all values i, j. Monty as a character corresponds to the predictions of quantum theory (9); he will assume C_1 is definitely a goat door since $|\langle \Phi_1 | \Psi_1 \rangle|^2 = 0$. This means the probabilities in (10) apply in this game, as well. Hence, the probability that Monty opens the prize door:

$$P(\text{opens prize door}) = \sum_{i=k\neq j} P(A_i \cap B_j \cap C_k) = \frac{1}{12} + \frac{2q}{3}. \tag{16}$$

This implies that the probability that Monty opens a goat door is $(11/12) - (2q/3)$. The probability of winning if always sticking and that Monty opens a goat door is:

$$\sum_{i=j\neq k} P(A_i \cap B_j \cap C_k) = \frac{1}{4}. \tag{17}$$

From this, we compute:

$$P(\text{win if stick} \,|\, \text{opens goat door}) = \frac{3}{11 - 8q}. \tag{18}$$

If a switching strategy is adopted, then:

$$\sum_{i=l\neq j\neq k} P(A_i \cap B_j \cap C_k \cap D_j) = \frac{1}{3} - \frac{q}{3}, \tag{19}$$

$$P(\text{win if switch} \,|\, \text{opens goat door}) = \frac{4 - 4q}{11 - 8q}. \tag{20}$$

Thus, the probabilities depend on whether the game is a ψ-ontic or ψ-epistemic game. For value $q = 1/4$, we can calculate that $P(\text{win if switch} \,|\, \text{opens goat door}) = P(\text{win if stick} \,|\, \text{opens goat door})$; hence, for certain ψ-epistemic games, switching offers no advantage.

7. Quantum Teleportation

Comparing a ψ-ontic game to a ψ-epistemic game, Monty opens the prize door less often. This corresponds to certain probabilities in the PBR proof being zero; some work on the experimental tests [22,46–49] of PBR discuss this exact zero probability as an experimental difficulty. Through our game, we provide another viewpoint: the difference in the probabilities of winning conditioned that a goat door is opened is simply different for the two physical scenarios. This may provide insights into alternative experimental designs to test PBR. A further application of these ideas can be extended into the realm of quantum information protocols such as teleportation.

Consider the standard protocol [43]. Alice wants to send some unknown state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ to Bob. They each possess a member of the Bell state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The initial state is $|\psi\rangle \otimes |\beta_{00}\rangle$. Alice applies a CNOTgate to her qubits, followed by a Hadamard gate to her first qubit. The resulting state can be written as:

$$\frac{1}{2} \Big(|00\rangle \, (\alpha |0\rangle + \beta |1\rangle) + |01\rangle \, (\alpha |1\rangle + \beta |0\rangle)$$
$$+ |10\rangle \, (\alpha |0\rangle - \beta |1\rangle) + |11\rangle \, (\alpha |1\rangle - \beta |0\rangle) \Big). \tag{21}$$

When Alice measures her qubits, she gets one of the results on the left in Equation (22). Bob would then apply the corresponding Pauli operator on his qubit to obtain $|\psi\rangle$:

$$
\begin{aligned}
00 &\rightarrow \text{Does nothing,} \\
01 &\rightarrow \text{Applies } \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|, \\
10 &\rightarrow \text{Applies } \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|, \\
11 &\rightarrow \text{Applies } \sigma_z\sigma_x.
\end{aligned} \tag{22}
$$

Bob receives the two bits from Alice in (22) through a classical channel. This protocol has been extended to probabilistic cases [50–52] and noisy cases [53–57].

7.1. Monty Hall Teleportation

For our first application, we want to modify the standard teleportation protocol into a Monty Hall game. Alice can be viewed as Monty and Bob as the contestant. The four doors are respectively labelled $(00, 01, 10, 11)$. This coincides with Alice's possible measurement results in (22); the prize door is Alice's actual result, whose bits we denote ab, and what Bob would need to get is the desired state $|\psi\rangle$. The contestant's initial choice of door would be equivalent to what Bell state was used at the start of the protocol. In this modification, the contestant is allowed to choose any of the four doors $(00, 01, 10, 11)$, which we denote xy. This event coincides with using Bell state:

$$
|\beta_{xy}\rangle = \frac{1}{\sqrt{2}}(|0\rangle |y\rangle + (-1)^x |1\rangle |\bar{y}\rangle), \tag{23}
$$

where $\bar{y}$ is the negation of y. As an example, if the contestant chooses door 01, then a way to implement this is that Bob applies the operator $(\sigma_0 \otimes \sigma_z) |\beta_{00}\rangle = |\beta_{01}\rangle$ and communicates that to Alice; the last step would be analogous to Monty being aware of what door the contestant chooses. In this modified protocol, the initial state is $|\psi\rangle |\beta_{xy}\rangle$. After Alice applies a CNOT gate to her qubits followed by a Hadamard gate, the resulting state is:

$$
\begin{aligned}
\frac{1}{2}\Big(&|00\rangle\,(\alpha |y\rangle + \beta(-1)^x |\bar{y}\rangle) + |01\rangle\,(\alpha(-1)^x |\bar{y}\rangle + \beta |y\rangle) \\
&+ |10\rangle\,(\alpha |y\rangle - \beta(-1)^x |\bar{y}\rangle) + |11\rangle\,(\alpha(-1)^x |\bar{y}\rangle - \beta |y\rangle)\Big).
\end{aligned} \tag{24}
$$

At this step, Alice measures her qubits to get her result. If Alice's result is $ab = xy$, meaning it coincides with the Bell state used $|\beta_{xy}\rangle$, then Bob has to do nothing, and he has the desired state $|\psi\rangle$ (the exception is if the initial Bell state used was $|\beta_{11}\rangle$ in which case Bob has to apply operator $(-\sigma_0)$ to get $|\psi\rangle$ if the result is 11). This is why the contestant's initial choice relates to the Bell state used.

In this Monty Hall protocol, Alice sends Bob two bits as in (22) with the following modification: she chooses two bits denoted cd (i.e., goat door) that are not xy (i.e., contestant's initial choice) and are not ab (i.e., prize door). Should Bob do nothing or apply one of the possible operators (which depend on what Bell state was used) to get $|\psi\rangle$, i.e., should the contestant stick or switch?

To answer this, let B_{xy} be the door chosen by the contestant. For this example, assume we use $|\beta_{00}\rangle$; hence, $P(B_{00}) = 1$. Let A_{ab} be the prize door, and due to the Born probabilities, we have $P(A_{ab}) = 1/4$. Let C_{cd} be the goat door opened by Monty, whose probabilities, from the protocol description, work out as:

$$
P(C_{cd} \mid B_{00} \cap A_{ab}) = \begin{cases} \frac{1}{3}, & \text{if } 00 = ab \neq cd, \\ \frac{1}{2}, & \text{if } 00 \neq ab \neq cd, \\ 0, & \text{otherwise.} \end{cases} \tag{25}
$$

If Bob always does nothing (i.e., stick strategy), then:

$$P(\text{win if stick}) = \sum_{ab=00\neq cd} P(A_{ab} \cap B_{00} \cap C_{cd}) = \frac{2}{8}. \tag{26}$$

Suppose Bob decides to always apply one of the two operators (i.e., switch strategy). Then, there is one of two possibilities, which we denote ef, and given it is a random choice, each occur with probability $1/2$. Let D_{ef} represent that door, and $P(\text{win if switch})$ is:

$$\sum_{ab=ef\neq cd\neq 00} P(A_{ab} \cap B_{00} \cap C_{cd} \cap D_{ef}) = \frac{3}{8}. \tag{27}$$

This means Bob should apply one of the two operators (switch) rather than do nothing (stick) to get state $|\psi\rangle$.

7.2. Unreliable Teleportation

For our second application, consider the standard teleportation protocol with the following unreliability: one of the two bits (either the first or second) Alice sends to Bob in (22) is received, but the other is lost; each event occurs with probability $1/2$. If the initial Bell state is $|\beta_{00}\rangle$ and Alice's result is 00, then Bob can do nothing. However, in this scenario, if Bob receives the single bit as 1, then the possible options are $01, 10,$ or 11; in this case, he should apply one of the operators (switch). If Bob receives bit 0, then his options are $00, 01, 10$. Should he stick (to 00) or switch (to 01 or 10)? To answer this, let us use the notation developed.

We have $P(B_{00}) = 1$ and $P(A_{ab}) = 1/4$. Let d in C_d be the single bit received by Bob; based on the scenario described above, we have $P(C_0 \mid B_{00} \cap A_{00}) = 1$, $P(C_0 \mid B_{00} \cap A_{01}) = 1/2$, and $P(C_0 \mid B_{00} \cap A_{10}) = 1/2$. We can compute the probability that Bob receives bit 0:

$$P(\text{received bit } 0) = \sum_{ab\neq 11} P(C_0 \cap B_{00} \cap A_{ab}) = \frac{1}{2}. \tag{28}$$

If Bob decides to always do nothing, then this would be like a sticking strategy. The probability that bit 0 is received and Bob wins by sticking is $P(A_{00} \cap B_{00} \cap C_0) = 1/4$. Hence, we can compute the conditional probability:

$$P(\text{win if stick} \mid \text{received bit } 0) = \frac{1/4}{1/2} = \frac{1}{2}. \tag{29}$$

If an always switching strategy is adopted, then there are two possibilities (01 or 10), each occurring with probability $1/2$. In this case, the probability of winning if switched and bit 0 is received is $P(A_{01} \cap B_{00} \cap C_0 \cap D_{01}) + P(A_{10} \cap B_{00} \cap C_0 \cap D_{10}) = 1/8$. With that, we compute,

$$P(\text{win if switch} \mid \text{received bit } 0) = \frac{1/8}{1/2} = \frac{1}{4}. \tag{30}$$

It is an advantage to stick, i.e., Bob should do nothing. This strategy may be used as an error correcting design for reliability issues in practical quantum networks [58,59].

8. Conclusions

There are various reasons for our gamification. These motivations are predicated on the work regarding Bell nonlocality and its nonlocal games. To be more precise, in one of the standard review papers on Bell's inequalities [2], it was emphasized, and we quote "Bell inequalities are also referred to as nonlocal games or sometimes simply as games. Looking at Bell inequalities through the lens of games often provides an intuitive understanding of their meaning. Such games enjoy a long history in computer science where they are known as interactive proof systems; see Condon (1989) for an

early survey. More recently, they have also been studied in the quantum setting, under the name of interactive proof systems with entanglement...” Therefore, we suggest that providing a gamification of yet another quantum foundation result advances this already established sub-field. Moreover, we hope our gamification may allow for non-trivial results in future work analogous to interactive proof systems for Bell's nonlocal games. The latter results contribute to this interdisciplinary field of quantum information science.

Crucial to the PBR theorem are non-orthogonal quantum states and their antidistinguishability. We reformulated the PBR theorem into a Monty Hall game by appropriately capturing this property of antidistinguishability. Furthermore, we applied these ideas to the quantum teleportation protocol. Future work may entail extending the Monty Hall concepts via antidistinguishability to quantum cryptography where non-orthogonal states play a central role [43]. Such investigations may lead to developing variations of the quantum key distribution where the sifted key may be acquired through a Monty Hall "switch or stick" procedure. Our work also involved utilizing the Monty Hall concepts to develop an error correcting strategy for unreliable teleportation due to bit loss. Given that the long range quantum key distribution is facing technological barriers due to photon loss [60], it may be useful to investigate whether an analogous Monty Hall error correcting strategy could be employed in such a cryptographic domain. On a more general line of thought, a quantum protocol involves the interplay of both a quantum channel and a classical channel. Hence, it would be interesting to observe whether developing advantages on the classical channel, such as a Monty Hall technique, could provide a new set of non-trivial quantum protocols that may have been overlooked given that, so far, the primary emphasis has been on the quantum channel. This work certainly alludes to such exciting possibilities.

References

1. Bell, J.S. On the Einstein–Podolsky–Rosen paradox. *Phys. Phys. Fiz.* **1964**, *1*, 195–200. [CrossRef]
2. Brunner, N.; Cavalcanti, D.; Pironio, S.; Scarani, V.; Wehner, S. Bell nonlocality. *Rev. Mod. Phys.* **2014**, *86*, 419. [CrossRef]
3. Brunner, N.; Linden, N. Connection between Bell nonlocality and Bayesian game theory. *Nat. Commun.* **2013**, *4*, 2057. [CrossRef]
4. Pappa, A.; Kumar, N.; Lawson, T.; Santha, M.; Zhang, S.; Diamanti, E.; Kerenidis, I. Nonlocality and conflicting interest games. *Phys. Rev. Lett.* **2015**, *114*, 020401. [CrossRef]
5. Almeida, M.L.; Bancal, J.; Brunner, N.; Acín, A.; Gisin, N.; Pironio, S. Guess your neighbor's input: A multipartite nonlocal game with no quantum advantage. *Phys. Rev. Lett.* **2010**, *104*, 230404. [CrossRef]
6. Coles, P.J.; Berta, M.; Tomamichel, M.; Wehner, S. Entropic uncertainty relations and their applications. *Rev. Mod. Phys.* **2017**, *89*, 015002. [CrossRef]
7. Coles, P.J.; Katariya, V.; Lloyd, S.; Marvian, I.; Wilde, M.M. Entropic energy-time uncertainty relation. *Phys. Rev. Lett.* **2019**, *122*, 100401. [CrossRef] [PubMed]
8. Eisert, J.; Wilkens, M.; Lewenstein, M. Quantum games and quantum strategies. *Phys. Rev. Lett.* **1999**, *83*, 3077. [CrossRef]
9. Benjamin, S.C.; Simon, C.; Hayden, P.M. Multiplayer quantum games. *Phys. Rev. A* **2001**, *64*, 030301. [CrossRef]
10. Khan, F.S.; Solmeyer, N.; Balu, R.; Humble, T.S. Quantum games: A review of the history, current state, and interpretation. *Quantum Inf. Process.* **2018**, *17*, 309. [CrossRef]

11. Rodriguez, A.; Mendes, B. *Probability, Decisions and Games: A Gentle Introduction Using R*; John Wiley & Sons: Hoboken, NJ, USA, 2018.

12. Rosenthal, J.S. Monty Hall, Monty fall, Monty crawl. *Math. Horiz.* **2008**, *16*, 5–7. [CrossRef]

13. Gill, R. Monty Hall problem. *Int. Encycl. Stat. Sci.* **2010**, 858–863. Available online: https://www.math. leidenuniv.nl/~gill/mhp-statprob.pdf (accessed on 31 December 2019).

14. Lucas, S.; Rosenhouse, J.; Schepler, A. The Monty Hall problem, reconsidered. *Math. Mag.* **2009**, *82*, 332–342. [CrossRef]

15. Li, C.; Zhang, Y.; Huang, Y.; Guo, G. Quantum strategies of quantum measurements. *Phys. Lett. A* **2001**, *280*, 257–260. [CrossRef]

16. Flitney, A.P.; Abbott, D. Quantum version of the Monty Hall problem. *Phys. Rev. A* **2002**, *65*, 062318. [CrossRef]

17. D'Ariano, G.M.; Gill, R.D.; Keyl, M.; Kümmerer, B.; Maassen, H.; Werner, R.F. The quantum Monty Hall problem. *Quant. Inf. Comput.* **2002**, *2*, 355–466.

18. Khan, S.; Ramzan, M.; Khan, M.K. Quantum Monty Hall problem under decoherence. *Commun. Theor. Phys.* **2010**, *54*, 47. [CrossRef]

19. Kurzyk, D.; Glos, A. Quantum inferring acausal structures and the Monty Hall problem. *Quantum Inf. Process.* **2016**, *15*, 4927–4937. [CrossRef]

20. Zander, C.; Casas, M.; Plastino, A.; Plastino, A.R. Positive operator valued measures and the quantum Monty Hall problem. *An. Acad. Bras. Ciênc.* **2006**, *78*, 417–422. [CrossRef]

21. Paul, S.; Behera, B.K.; Panigrahi, P.K. Playing quantum Monty Hall game in a quantum computer. *arXiv* **2019**, arXiv:1901.01136.

22. Pusey, M.F.; Barrett, J.; Rudolph, T. On the reality of the quantum state. *Nat. Phys.* **2012**, *8*, 475–478. [CrossRef]

23. Harrigan, N.; Spekkens, R.W. Einstein, incompleteness, and the epistemic view of quantum states. *Found. Phys.* **2010**, *40*, 125–157. [CrossRef]

24. Lewis, P.G.; Jennings, D.; Barrett, J.; Rudolph, T. Distinct quantum states can be compatible with a single state of reality. *Phys. Rev. Lett.* **2012**, *109*, 150404. [CrossRef]

25. Schlosshauer, M.; Fine, A. Implications of the Pusey–Barrett–Rudolph quantum no-go theorem. *Phys. Rev. Lett.* **2012**, *108*, 260404. [CrossRef] [PubMed]

26. Aaronson, S.; Bouland, A.; Chua, L.; Lowther, G. ψ-epistemic theories: The role of symmetry. *Phys. Rev. A* **2013**, *88*, 032111. [CrossRef]

27. Patra, M.K.; Pironio, S.; Massar, S. No-go theorems for ψ-epistemic models based on a continuity assumption. *Phys. Rev. Lett.* **2013**, *111*, 090402. [CrossRef] [PubMed]

28. Schlosshauer, M.; Fine, A. No-go theorem for the composition of quantum systems. *Phys. Rev. Lett.* **2014**, *112*, 070407. [CrossRef]

29. Mansfield, S. Reality of the quantum state: Towards a stronger ψ-ontology theorem. *Phys. Rev. A* **2016**, *94*, 042124. [CrossRef]

30. Leifer, M.S. Is the quantum state real? An extended review of ψ-ontology theorems. *Quanta* **2014**, *3*, 67–155. [CrossRef]

31. Jennings, D.; Leifer, M. No return to classical reality. *Contemp. Phys.* **2016**, *57*, 60–82. [CrossRef]

32. Leifer, M.S.; Pusey, M.F. Is a time symmetric interpretation of quantum theory possible without retrocausality? *Proc. R. Soc. A Math. Phys. Eng. Sci.* **2017**, *473*, 20160607. [CrossRef]

33. Leifer, M.S. ψ-epistemic models are exponentially bad at explaining the distinguishability of quantum states. *Phys. Rev. Lett.* **2014**, *112*, 160404. [CrossRef] [PubMed]

34. Barrett, J.; Cavalcanti, E.G.; Lal, R.; Maroney, O.J.E. No ψ-epistemic model can fully explain the indistinguishability of quantum states. *Phys. Rev. Lett.* **2014**, *112*, 250403. [CrossRef] [PubMed]

35. Branciard, C. How ψ-epistemic models fail at explaining the indistinguishability of quantum states. *Phys. Rev. Lett.* **2014**, *113*, 020409. [CrossRef] [PubMed]

36. Montina, A. Epistemic view of quantum states and communication complexity of quantum channels. *Phys. Rev. Lett.* **2012**, *109*, 110501. [CrossRef]

37. Montina, A. Communication complexity and the reality of the wave function. *Mod. Phys. Lett. A* **2015**, *30*, 1530001. [CrossRef]

38. Perry, C.; Jain, R.; Oppenheim, J. Communication tasks with infinite quantum-classical separation. *Phys. Rev. Lett.* **2015**, *115*, 030504. [CrossRef]

39. Bandyopadhyay, S.; Jain, R.; Oppenheim, J.; Perry, C. Conclusive exclusion of quantum states. *Phys. Rev. A* **2014**, *89*, 022336. [CrossRef]

40. Arunachalam, S.; Molina, A.; Russo, V. Quantum hedging in two-round prover-verifier interactions. *arXiv* **2017**, arXiv:1310.7954.

41. Myrvold, W.C. ψ-ontology result without the Cartesian product assumption. *Phys. Rev. A* **2018**, *97*, 052109. [CrossRef]

42. Bennett, C.H.; Brassard, G.; Crépeau, C.; Jozsa, R.; Peres, A.; Wootters, W.K. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **1993**, *70*, 1895. [CrossRef]

43. Nielsen, M.A.; Chuang, I.L. *Quantum Computation and Quantum Information*; Cambridge University Press: Cambridge, UK, 2010.

44. Caves, C.M.; Fuchs, C.A.; Schack, R. Conditions for compatibility of quantum-state assignments. *Phys. Rev. A* **2002**, *66*, 062111. [CrossRef]

45. Heinosaari, T.; Kerppo, O. Antidistinguishability of pure quantum states. *J. Phys. A Math. Theor.* **2018**, *51*, 365303. [CrossRef]

46. Nigg, D.; Monz, T.; Schindler, P.; Martinez, E.A.; Hennrich, M.; Blatt, R.; Pusey, M.F.; Rudolph, T.; Barrett, J. Can different quantum state vectors correspond to the same physical state? An experimental test. *New J. Phys.* **2015**, *18*, 013007. [CrossRef]

47. Miller, D.J. Alternative experimental protocol to demonstrate the Pusey–Barrett–Rudolph theorem. *Phys. Rev. A* **2013**, *87*, 014103. [CrossRef]

48. Ringbauer, M.; Duffus, B.; Branciard, C.; Cavalcanti, E.G.; White, A.G.; Fedrizzi, A. Measurements on the reality of the wavefunction. *Nat. Phys.* **2015**, *11*, 249–254. [CrossRef]

49. Liao, K.; Zhang, X.; Guo, G.; Ai, B.; Yan, H.; Zhu, S. Experimental test of the no-go theorem for continuous ψ-epistemic models. *Sci. Rep.* **2016**, *6*, 26519. [CrossRef]

50. Li, W.; Li, C.; Guo, G. Probabilistic teleportation and entanglement matching. *Phys. Rev. A* **2000**, *61*, 034301. [CrossRef]

51. Lu, H.; Guo, G. Teleportation of a two-particle entangled state via entanglement swapping. *Phys. Lett. A* **2000**, *276*, 209–212. [CrossRef]

52. Agrawal, P.; Pati, A.K. Probabilistic quantum teleportation. *Phys. Lett. A* **2002**, *305*, 12–17. [CrossRef]

53. Fortes, R.; Rigolin, G. Fighting noise with noise in realistic quantum teleportation. *Phys. Rev. A* **2015**, *92*, 012338. [CrossRef]

54. Fortes, R.; Rigolin, G. Probabilistic quantum teleportation in the presence of noise. *Phys. Rev. A* **2016**, *93*, 062330. [CrossRef]

55. Knoll, L.T.; Schmiegelow, C.T.; Larotonda, M.A. Noisy quantum teleportation: An experimental study on the influence of local environments. *Phys. Rev. A* **2014**, *90*, 042332. [CrossRef]

56. Carlo, G.G.; Benenti, G.; Casati, G. Teleportation in a noisy environment: A quantum trajectories approach. *Phys. Rev. Lett.* **2003**, *91*, 257903. [CrossRef] [PubMed]

57. Kumar, D.; Pandey, P.N. Effect of noise on quantum teleportation. *Phys. Rev. A* **2013**, *68*, 012317. [CrossRef]

58. Simon, C. Towards a global quantum network. *Nat. Photonics* **2017**, *11*, 678–680. [CrossRef]

59. Ren, J.; Xu, P.; Yong, H.; Zhang, L.; Liao, S.; Yin, J.; Liu, W.; Cai, W.; Yang, M.; Li, L.; et al. Ground-to-satellite quantum teleportation. *Nature* **2017**, *549*, 70–73. [CrossRef]

60. Azuma, K.; Tamaki, K.; Lo, H. All-photonic quantum repeaters. *Nat. Commun.* **2015**, *6*, 6787. [CrossRef]