3 (Sem-6/CBCS) PHY HE 5

## 2024

## PHYSICS

(Honours Elective)

Paper: PHY-HE-6056

(Classical Dynamics)

Full Marks: 80

Time: Three hours

## The figures in the margin indicate full marks for the questions.

- 1. Answer the following questions: 1×10=10
  - (a) Write the necessary and sufficient condition for force F to be conservative.
  - (b) A system of 4 particles has 10 equations of constraints and requires 2 generalized coordinates. Are the constraints holonomic or non-holonomic?
    - (c) Write the expression for Hamiltonian of a free particle in spherical polar coordinates.

- (d) State one advantage of Lagrangian formulation over Newtonian formulation.
- (e) What is called gyro frequency? Write down its expression.
- (f) Write down the relativistic form of Newton's second law of motion.
- (g) What is the significance of Reynold's number?
- (h) Write down the Newton's law of viscous flow in streamline motion and hence define the coefficient of viscosity.
- (i) Express equation of continuity in terms of four current density vector.
- (j) Write down the Lorentz transformation equations of energy and momentum.
- 2. Answer the following questions:  $2 \times 5 = 10$ 
  - (a) "Magnetic field changes the velocity of a charged particle without changing its speed." Explain the statement.
  - (b) Show that Lagrangian and Newtonian equations of motion are equivalent.
  - (c) What are the different types of relativistic optical Doppler effects?

- (d) State and explain postulates of special theory of relativity.
- (e) A tube of radius r and length l is connected in series with another of radius  $\frac{r}{2}$  and length  $\frac{l}{4}$ . If the pressure across the two tubes taken together is p, deduce the pressures across the tubes separately.
- 3. Answer any four from the following questions: 5×4=20
  - (a) Show that the path of a charged particle in a uniform magnetic field, in general, is a helix. Under what condition is this path reduced to a circle? 4+1=5
  - (b) Derive Lagrange's equations of motion for a conservative system using D' Alembert's principle.
  - (c) What do you mean by the element of both old proper time?

    Using four vector expressions show that  $E^2 = p^2c^2 + m_0^2c^4$ , where symbols have their usual meanings.

    1+4=5
    - (d) Write brief notes on space-like and time-like intervals.

- (e) Express Lorentz transformations of space and time in four vector form.
- (f) (i) Using Euler-Lagrange equation prove that 'the shortest distance between two points in a plane is a straight line'.
  - (ii) State Hamilton's principle. 1
- 4. Answer the following questions: 10×4=40
  - (a) (i) Show that Hamiltonian H is a constant of motion if the Lagrangian L is not an explicit function of time.
    - (ii) Derive Hamilton's canonical equations and use these to obtain the equation of motion of a simple pendulum.

      4+4=8

Or

- (b) (i) What do you mean by stable and unstable equilibria? 2+2=4
  - (ii) Obtain Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium.

- (iii) Write the principle of virtual work in terms of independent generalized coordinates. 2
- (c) (i) The equation of the orbit of a particle under the action of a central force is  $r = 2a\cos\theta$ .

Show that the force F is inversely proportional to  $r^5$ .

- (ii) Prove the conservation of energy of a particle directly from its equation of motion in a central force field.
- (iii) The nature of orbit is determined by the value of its eccentricity

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{\mu k^2}}$$

where symbols have their usual meanings.

Mention the various special cases depending upon the values of E and  $\varepsilon$ .

- (d) (i) A proton with initial velocity of  $5 \times 10^6 \, ms^{-1}$  passes through an electric field (transverse) of  $200 \, volt/cm$ . Calculate the transverse deflection in travelling a distance of  $1 \, m$ .
  - (ii) Obtain equations of motion of a system of coupled simple pendulums by setting Lagrangian of the entire system.
- (e) (i) Explain the concept of twin paradox with the help of spacetime diagram.
  - (ii) Draw a neat diagram of light cones indicating past and future. Show the world lines in it.

Can the tangent to the world line of a massive particle at a point has an angle equal to or more than 45°? Explain.

3+2=5

- What are called cyclic or ignorable coordinates? If a system undergoes translatory motion along a cyclic generalized coordinate  $q_k$ , will the Lagrangian of the system be affected?
  - (ii) Obtain the Lagrange's equation of motion for an electrical circuit comprising an inductance L and capacitance C. The capacitor is charged to q coulombs and the current flowing in the circuit is i amperes.
  - (iii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four-dimensional Minkowski space.
- (g) (i) State length centraction and time dilation. How are the phenomenon of length contraction and time dilation interpreted on space-time diagram? 2+(3+3)=8
  - (ii) Calculate the velocity which 1 amu of mass will have, if it had a kinetic energy 3 times the rest mass energy.

- capillary tube is  $V = \frac{P\pi r^4}{8\eta l}$  with usual notations. Deduce the relation stating clearly the conditions under which it holds.
- bus 1 Discuss the corrections to be applied to Poiseuille's equation.

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