2019

STATISTICS

(Major)

Paper : 4.1

(Mathematical Methods—III and OR—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) State Cayley-Hamilton theorem.

(b) Define basis of a vector space.

(c) What are eigenvalues?

(d) When a distribution has fewer than $m+n-1$ allocation, it is degenerate.

(Write True or False)

(e) When in a transportation problem the total availability from all origins is equal to the total demand at all the destinations, the problem is balanced.

(Write True or False)

(f) What is convex set?

(g) Write an assumption of linear programming problem.

2. Answer the following questions : 2×4=8

(a) Show that the vectors $X_1 = (1, 2, 4)$ and $X_2 = (3, 6, 12)$ are linearly dependent.

(b) Give the procedure for mathematical formulation of a linear programming problem.

(c) If A is non-singular matrix, then prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

(d) Show that intersection of two convex sets is also a convex set.

3. Answer any *three* of the following questions : 5×3=15

(a) Prove that all the extreme points are boundary points but the converse is not necessarily true.

(b) What is transportation problem? Show that it can be considered as an LPP.

(c) What do you mean by linear programming problem? Explain.

(d) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

(e) How can a linear programming problem be solved by graphical method?

4. Answer any *three* of the following questions : 10×3=30

(a) Determine the characteristic vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 10

(b) *(i)* Prove that the convex polyhedron is a convex set. 5

(ii) Prove that in E^2, the set

$$X = \{(x, y) / x^2 + y^2 \leq 4\}$$

is a convex set. 5

(c) *(i)* Solve graphically the following LPP : 5

Maximize $Z = 2x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 1$$
$$3x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

(ii) Give the computational procedure for simplex method in linear programming. 5

(d) *(i)* Prove that a necessary and sufficient condition for the existence of a feasible solution to an $m \times n$ transportation problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

where a_i and b_j denote the availability and requirement at i-th origin and j-th destination respectively. 5

(ii) Explain North-West Corner Rule for finding an initial basic feasible solution for a transportation problem. 5

(e) *(i)* Prove that the non-empty subset of any linearly independent set of vectors is linearly independent. 5

(ii) Show that the characteristic roots of a triangular matrix are just the diagonal elements of the matrix. 5

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